Pressure Drop for In-tube Supercritical CO\textsubscript{2} Cooling: 
Comparison of Correlations and Validation

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Abstract

The pressure drop for supercritical CO\textsubscript{2} cooling in gas cooler tubes for heat pump applications has gained interest due to great variation in thermo-physical properties (specially in the pseudo-critical region). This study presents a detailed comparison of pressure drop correlations available for supercritical CO\textsubscript{2} cooling in mini and micro tubes. For the comparison, a general numerical model has been established for the gas cooler. The configuration considered here is a counter-flow, tube-in-tube type with CO\textsubscript{2} gas flowing in inner tube and the secondary fluid (coolant) water flowing in the annulus.

All the recommended pressure drop correlations developed for other fluids or correlations specially developed for CO\textsubscript{2} cooling are based on the limited set of dimensions and operating conditions. So the correlations are required to be validated for other dimensions and conditions, because a more general correlation is needed for the design of an entire heat exchanger. A test facility has been developed to investigate the supercritical carbon dioxide cooling in tubes with water as coolant. Subsequently the correlations have been validated with the tested pressure drop data generated for convective gas cooling. This study is expected to help the design engineer by recommending an appropriate pressure drop correlation for actual gas cooler design to be employed in transcritical CO\textsubscript{2} cycle based cooling-heating systems.

Keywords: Supercritical CO\textsubscript{2} in-tube cooling, pressure drop correlations, comparison, experimental validation.

Nomenclature

\begin{tabular}{ll}
$c_p$ & Specific heat capacity, kJ kg\textsuperscript{-1} K\textsuperscript{-1} \\
$D$ & Diameter, m \\
$f$ & Friction factor \\
$G$ & Mass velocity, kg m\textsuperscript{-2} s\textsuperscript{-1} \\
$h$ & Heat transfer coefficient, W m\textsuperscript{-2} K\textsuperscript{-1} \\
$k$ & Thermal conductivity, W m\textsuperscript{-1} K\textsuperscript{-1} \\
$L$ & Length, m \\
$Nu$ & Nusselt number \\
$Pr$ & Prandtl number \\
$q_w$ & Heat flux, kW m\textsuperscript{-2} \\
$Re$ & Reynolds number \\
$R_r$ & Relative roughness \\
$T$ & Temperature, K \\
$\Delta p$ & Pressure drop, bar \\
$\mu$ & Viscosity, N m\textsuperscript{-2} s \\
$\rho$ & Density, kg m\textsuperscript{-3} \\
\end{tabular}
1. Introduction

Within last decade, natural refrigerant CO\textsubscript{2} based transcritical cycle has gained special interest in refrigeration, heat pump and air-conditioning applications due to various advantages such as lower pressure ratio, higher refrigerant capacity, easy availability and superior heat transfer properties along with zero ODP and negligible GWP compared to synthetic refrigerants. Due to low critical temperature (31.1°C), supercritical heat rejection is taking place for CO\textsubscript{2} in the gas cooler instead of condenser for conventional cycle. The temperature profile of cooled CO\textsubscript{2} thus can be matched up with temperature profile of cooling medium with single-phase heat transfer such as water. This gives reduced thermodynamic losses in single-phase heating. Thus, single-phase heat transfer in gas cooler leads various application possibilities of transcritical CO\textsubscript{2} heat pumps in water heating and other simultaneous process heating and cooling applications such as in food and agricultural industries [1].

Optimum design of gas cooler is an important parameter for designing a CO\textsubscript{2} based refrigeration or heat pump system. Supercritical CO\textsubscript{2} gas cooling operates very close to the critical point of CO\textsubscript{2} which means large variation in the properties of CO\textsubscript{2} with temperature and precisely due to this it is very difficult to predict heat transfer characteristics of supercritical CO\textsubscript{2} during tube cooling process using various turbulent models. The great variation in thermo-physical properties (specially in the pseudo-critical region) cause the heat transfer coefficient and pressure drop of carbon dioxide to be greatly dependent on both the local temperature and the heat flux in gas cooler tubes. Hence detail study on supercritical CO\textsubscript{2} gas cooling pressure drop for different operating conditions such as pressure and mass flow rates, dimensional parameters is required for rigorously understanding the processes and optimum design of gas cooler.

Pitla et al. [2] and Fang [3] have reviewed the supercritical heat transfer and pressure drop characteristics of carbon dioxide in-tube flow primarily including effects of physical factors on supercritical heat transfer and friction factor correlations. Almost all the general single-phase pressure drop correlations available in the open literature are unsuitable near the pseudo-critical region due to abrupt variation of thermophysical properties. Hence, new or modified pressure drop correlation for in-tube supercritical CO\textsubscript{2} cooling is required to take care of the variation of properties in both radial and longitudinal direction of flow. Literature search shows very limited friction factor correlations based on the constant thermophysical properties both for smooth tube (Blasius’ equation & Filonenko’s equation [4]) and rough tube (Colebrook’s equation & Althul’s modified equation [3], Churchill’s equation [5]). Petrov & Popov [6] proposed the friction factor correlation for supercritical CO\textsubscript{2} cooling based on the numerical calculation. They have also proposed another correlation which was calculated basically for cooling of supercritical water [7]. Although there is no such correlation available, it was developed based on the experiment on the supercritical CO\textsubscript{2} cooling. To take care of the great variation of thermophysical properties, validation of available correlation is required. Recently few experimental validations on friction factor correlations for supercritical CO\textsubscript{2} cooling were conducted (Yoon et al. [8], Dang et al. [9], Son et al. [10]) but recommended correlations are not common. So, still need the validation of the correlations for other dimensions and flow rates, because we need a more general correlation for the design of an entire heat exchanger.

In the present work a detailed comparison of available friction factor correlations for the pressure drop calculation both in smooth tube and rough tube and validation with experimental results for supercritical carbon dioxide cooling in mini and micro tubes have been carried out. For the comparison, a general numerical model has been established for the gas cooler. The configuration considered here is a counter-flow, tube-in-tube type with CO\textsubscript{2} gas flowing in inner tube and the secondary fluid (coolant) water flowing in the annulus. A test facility has been developed to investigate the supercritical carbon dioxide cooling in tubes with water as coolant. Subsequently, pressure drop have been measured across the gas cooler for various operating conditions and the correlations have been validated.

2. Theoretical Analysis

Total pressure drop in the gas cooler can be calculated by,

$$\Delta p = \frac{G^2}{2\rho} \left( f_h \frac{L}{D} + \lambda \right)$$

(1)

Where, \(\lambda\) is the local pressure drop coefficient, whose recommended value is 1.2. Hydraulic drag, \(f_h\) [4] is given by, \(f_h = f + f_i\). Where, the inertia factor as in one dimensional approximation is expressed as:

$$f_i = \frac{8q_v}{Gc_p} \left[ -\frac{1}{\rho} \frac{\partial \rho}{\partial t} \right]_{\text{in}}$$

(2)

For incompressible fluid flow, the value of inertia factor is 0. So, for in-tube CO\textsubscript{2} cooling in supercritical region, for simplicity, it is recommended to neglect inertia factor; i.e., \(f_h = f\).
2.1 Friction factor correlations

Large numbers of friction factor correlations have been developed based on constant thermophysical property both for smooth tube and rough tube. Blasius equation and Filonenko’s equation are the widely used in friction factor calculation for turbulent flow in smooth tube and constant thermo-physical property [4]. Blasius equation is given by,

\[ f = 0.316 \frac{1}{2^{1/4}} \text{ for } \text{Re} \leq 2 \times 10^4 \]
\[ f = 0.184 \frac{1}{2^{1/5}} \text{ for } \text{Re} \geq 2 \times 10^4 \]  

(3)

Filonenko’s equation for friction coefficient is given by,

\[ f = [0.79 \ln(\text{Re}) - 1.64]^2 \]  

(4)

Above equation is only valid for smooth tube with fully turbulent flow (\( \text{Re} > 8 \times 10^3 \)).

Althul’s modified equation [3] for the friction factor for rough tube for the range of \( 3 \times \text{Re} > 8 \times 10^3 \) and \( \frac{\text{Re}}{\text{R}_n} \leq 0.001 \):

\[ f' = 0.11 (\text{R}_n + 68 / \text{Re})^{0.25} \]
\[ f = \begin{cases} f' & \text{if } f' \leq 0.018 \\ 0.0028 + 0.085 f' & \text{if } f' > 0.018 \end{cases} \]  

(5)

Churchill [5] proposed a more complicated equation for all flow regimes & all relative roughness, which agrees with Moody diagram:

\[ f = 8 \left( \frac{8}{\text{Re}} \right)^{12} + \left[ \frac{2.457 \ln \left( \frac{1}{(7/\text{Re})^{1/9} + 0.27 \text{R}_n} \right)}{\left( \frac{36530}{\text{Re}} \right)^{16/3}} \right]^{1/12} \]  

(6)

Thermophysical property variations in the cooling conditions at supercritical pressures significantly affect the pressure drop characteristics. No CO₂ specific experimental correlations are found for the friction factor in the cooling condition at supercritical pressures.

Petrov and Popov [6] solved governing equations numerically using the Prandtl mixing length of turbulence and proposed following formulas for the friction factor of CO₂ cooled in supercritical conditions in the range of \( \text{Re}_w = 1.4 \times 10^4 - 7.9 \times 10^5 \) and \( \text{Re}_b = 3.1 \times 10^4 - 8 \times 10^7 \):

\[ f = (1.82 \ln(\text{Re}_w) - 1.64)^{-2} \rho_w \left( \frac{\rho_w}{\rho_b} \right)^s \left( \frac{\mu_w}{\mu_b} \right)^s \]  

(7)

where, \( s = 0.023 \left( \frac{\rho_w}{G} \right)^{0.42} \)

Later in 1988, they [7] calculated the friction factor for cooling of supercritical water in range of \( \text{Re}_w < 2.0 \times 10^3 - 1.88 \times 10^5 \) and \( \text{Re}_m < 2.3 \times 10^4 - 2.03 \times 10^5 \) as follows:

\[ \frac{f}{f_{0m}} = \left( \frac{\rho_w}{\rho_m} \right)^{1/4} + 0.17 \left( \frac{\rho_w}{\rho_m} \right)^{1/3} \frac{f_{0w}}{f_{0m}} \]  

(8)

They claimed that this equation is also applicable for helium and carbon dioxide. The friction factors \( f_{0w} \) and \( f_{0m} \) are calculated by Filonenko’s equation at \( T_w \) and \( T_m \) respectively, and Filonenko’s equation is only used for the fully developed turbulent flow in smooth tubes. In order to extend the use of equations 7 and 8 to transitional regime and rough tube, Churchill’s equation 4 is recommended instead of Filonenko’s equation to calculate \( f_{0w} \) and \( f_{0m} \). Fang et al. [3] recommended that since the equation 8 is derived from calculation of water supercritical cooling; it is better to use equation 7 for supercritical CO₂ cooling.

2.2 Comparison of correlations

A simple computer code has been developed for comparison of various friction factor correlations, presented in the above section, for different combination of operating and dimensional parameters. For thermophysical and transport properties of CO₂, an exclusive property subroutine has been developed based on seminal work by Span abd Wagner [11], Vesovic et al. [12] and Feghour et al. [13] and incorporated in the code.

Fang et al. [3] have done a comparison study of Blasius equation (3), Filonenko’s equation (4), Althul’s modified equation (5) and Churchill equation (6) with respect to Re and showed that Blasius equation can be valid for \( \text{Re} < 1.5 \times 10^8 \) and Filonenko equation can be used for \( \text{Re} > 8 \times 10^7 \).

Figure 1 shows the variation of friction factor along the SS tube from inlet to outlet for in-tube CO₂ cooling. Counter-flow heat exchanger with inner tube diameters 4.75 mm/ 6.35 mm and outer tube diameters 10 mm/ 12 mm, has been assumed. Following input parameters have been taken for generated data: Gas cooling pressure = 100 bar, inlet and outlet temperature of CO₂ are 393 K and 308 K, mass flow rate of CO₂ is 0.04 kg/s, inlet and outlet temperatures of secondary fluid water are 298 K and 353 K. For the simulation work, the heat exchanger has been discretized to consider the lengthwise property variation and energy conservation equations have been applied to each
segment. For refrigerant side heat transfer calculation, Pitla et al. [14] correlation employing the mean Nusselt number concept based on a numerical and experimental study has been used to account for the property variation in the perpendicular direction of flow:

\[
Nu = \left( \frac{Nu_w + Nu_b}{2} \right) \frac{k_w}{k_b} \frac{h}{D} = \frac{Nu}{D} \quad (9)
\]

\[
Nu_w = \frac{(f/8)(Re-1000)Pr^{1/3}}{1.07 + 12.7(f/8)^{1/2}(Pr_0^{2/3} - 1)} \quad (10)
\]

Result shows that both the friction factors increase due to decrease in Reynolds number with the decrease in refrigerant temperature along the gas cooler. Petrov-Popov correlation for friction factor, which is based on the calculation of supercritical CO2 cooling considering the property variation overpredicts by about 7.5\% than the constant property based Filonenko’s equation.

3. Experimental Validation

3.1 Experimental methodology

To study the pressure drop characteristics in the gas cooler, the test facility for the prototype of a transcritical CO2 heat pump for simultaneous water cooling and heating, developed in IIT Kharagpur has been used. Figure 2 shows a design layout of the test facility and the Figure 3 shows the photograph of the prototype with accessories and instrumentation. Saturated or superheated vapour from separator (item 11) is compressed to high pressure through compressor (item 4) and the compressed hot CO2 gas is cooled as it flows through gas cooler (item 6). Then the cooled CO2 fluid is expanded to evaporation pressure through expansion device (item 9) and the resulting two-phase CO2 passes through evaporator (item 10) to give the cooling effect. Two-phase, saturated vapour or superheated vapour of CO2 exits from the evaporator and enters the separator. A receiver (item 7) is used between gas cooler and expansion device to control the pressures. A Coriolis mass flow meter (item 3) is installed between the separator and the compressor to measure the mass flow rate and temperature of CO2 vapour entering the compressor. Two Swagelok safety valves (item 2) are used in both low and high pressure sides to control the higher limit of pressure. Four pressure gauges (item 5) have been used in different locations. One differential pressure gauge (item 8) is used to measure the refrigerant side pressure drop in the gas cooler. CO2 cylinder (item 1) is used for external charging of CO2. A W-bend is provided before compressor to provide superheating of CO2 vapour, if required. A temperature-controlled bath is used in which W-bend is immersed. But this was never used, because the outlet of evaporator was already sufficiently superheated at operating conditions. However, this provision for heating the refrigerant could be requisitioned during cooler water inlet ambient temperatures in winter. A separate air-cooled condensing unit is used to supply water at required temperature and flow rate to the gas cooler. For water inlet to evaporator at required flow rate and temperature, a separate heating unit was used (not shown in figure). Thermocouples were fitted in different sections to measure various temperatures; the sensors were subsequently connected to the data acquisition system (DAS), which is interfaced with a computer as shown in Figure 4.

A Dorin CO2 compressor (model TCS113) was chosen for the experimental investigation. On the basis of minimum and maximum pressure ratios of 80/50 and 120/26 (bar/bar), respectively, a Swagelok integral bonnet needle valve (model SS-1RS4) was used as the expansion device. The separator and receiver were designed for a total volumetric capacity of 8 L and 2 L, respectively. A condensing unit including a fan and a storage tank was employed for a heat transfer rate of 6 kW to cool the warm water to its initial temperature at the inlet to the gas cooler. A water bath with heater and pump was incorporated in the evaporator to supply
water at constant temperature and flow rate, so that a cooling capacity of 3.5 kW can be obtained. The evaporator and the gas cooler are counter-flow tube-in-tube heat exchangers, where CO₂ flows in the inner tube and water in the outer annulus. Once fabricated, both the heat exchangers were tested up to 120 bar pressure for leak detection and pressure sustainability.

Due to single-phase flow in the gas cooler, pressure drop is very low compared to that of evaporator for the same conditions and the same diameter (as discussed in chapter 5). For 1/4 inch OD, the maximum pressure drop is around 1 bar accompanied by excellent heat transfer rate as is evident from the simulation. Hence for the present system design condition, 1/4 inch (6.35 mm) OD was chosen for the gas cooler. The tube thickness was taken as 0.8 mm (leading to 4.75 mm ID), which is sufficient to withstand the expected refrigerant pressure in the gas cooler. For the water side (tube annulus), standard stainless tube of 12 mm OD with 1 mm thickness (10 mm ID) was taken. Mass flow rate of water was in the range of 1.2-3.2 L/min for design conditions. The cross-sectional area ratio of water to refrigerant was 2.64. Total designed heat transfer length for the gas cooler was taken as 14 m. Figure 5 shows the layout of tube-in-tube counter-flow gas cooler, which was designed for the experiment. Refrigerant flows through inner tube whereas the water flows through the annulus. For the sake of simplicity of fabrication, only two rows were considered. Gas cooler contains 14 parallel segments in two rows, each 1 m long, where the refrigerant tubes are connected by 180° circular bends having the same diameter and the water tubes are connected by 90° straight tubes of 9.5 mm OD. Sufficient gap between the two parallel segments was maintained in fabrication for proper insulation and handling. The gas cooler fabrication closely followed that of the evaporator and similar practices were implemented in both the fabrication processes. After the fabrication, the gas cooler was tested up to a pressure of 120 bar for leak detection and pressure sustainability. The fabricated gas cooler, as shown in Figure 6, has an effective total heat transfer length of about 13.6 m. The gas cooler was properly insulated by glassfibre insulation to reduce the heat transfer with the ambient. The thermocouples were connected to each segment for detailed study of heat transfer through the gas cooler.
After leak testing of the individual components, they were assembled with suitable instruments. The assembled system was again tested for leaks under high pressure. All the measuring devices were calibrated as per standard procedure (for example, thermocouple by thermostatic bath). Then the system was purged and then charged with CO₂. Before switching on the compressor, condensing unit for the gas cooler and heating unit for the evaporator were started to stabilize the temperature in the system and total refrigerant charge present in the system was estimated. After recording initial reading and starting the data scan of the DAS, the compressor was switched on. Control of discharge pressure was achieved by simultaneous control of the total mass of CO₂ in the system and degree of opening of the expansion device. The total refrigerant mass in the system was controlled by adding CO₂ from a high pressure cylinder or by venting it through the safety valve. The operating parameters were varied independently following a test matrix. Temperatures were monitored at all required locations. The principal system performance parameters under steady state, namely, power input to the compressor, refrigeration capacity and cooling output in evaporator, heat rejection rate of refrigerant and heating output in gas cooler, cycle COP and actual COP have been computed from the measured data. Detailed calculation procedures and experimental uncertainty analyses have been reported elsewhere [15]. Repeatability tests for two sets of operating parameters showed that most of the data points for cooling COPs are within the uncertainty ranges (± 6%) of the test loop measurements in both cases.

The Swagelok differential pressure gauge has been used for measurement of the pressure drop in the gas cooler, which has the range of 0–4 bar with the accuracy of ±1.5%. The pressure drop has been calculated for the different operating conditions.

3.2 Results and discussions

Only Petrov-Popov equation has been developed based on the numerical analysis of supercritical CO₂ cooling and this equation has been validated with the experimental data. Although compressibility affect part is neglected due to use of mini-tube as it is important for micro-tube. Using the operating condition recorded in the experiment, the pressure drop has been calculated using the numerical model presented above by using Equations (1) & (7). Figure 7 shows the measured and predicted trend of pressure drop using the correlation used for simulation for different pressure and mass flow rates of CO₂. Pressure drop due to tube bending has also been considered for estimating the predicted data. Comparison shows that the measured values are higher (up to 55% deviation) than predicted values. One reason may be due to tube roughness, which was neglected for pressure drop estimation for the predicted data. Use of Churchill equation instead of Filonenko equation gives less deviation.

3.3 Other experimental validations

Based on the experimental data, Yoon et al. [8] recommended Blasius’ correlation (Equation 3) for the prediction of pressure drop in the supercritical region of carbon dioxide. Previous experimental results by Dang et al. [9] showed that Filonenko’s equation (Equation 4) gives reasonable prediction under small diameter tube conditions. Son et al. [10] conducted test for horizontal tube and compared the experimental data with the prediction by Blasius and Petrov–Popov [4] correlations. The correlation, which is proposed by Blasius agreed quite well with the experimental data. The mean deviation of the measured and predicted pressure drop is 4.6%, whereas for Petrov–Popov mean deviation is 64% Therefore, they recommended Blasius’s correlation for the prediction of pressure drop during gas cooling process of CO₂ in the supercritical region.

4. Conclusion

Detailed comparison of friction factor correlations to calculate the pressure drop for supercritical carbon dioxide cooling in mini and micro tubes has been done by establishing a simulation code for counter-flow, tube-in-tube gas cooler. A test facility has been developed to investigate the supercritical carbon dioxide cooling in tubes with water as coolant and the pressure drop correlation has been validated with the experimental data generated. Comparison clearly shows that Blasius equation can be valid for \( Re < 1.5 \times 10^5 \) and Filonenko equation can be used for \( Re > 8 \times 10^3 \) and Petrov-Popov correlation for friction factor overpredicts by about 7.5% than the constant property based Filonenko’s equation. The variation of friction factor along the tube is not significant. The absolute value of inertia drag, which is negative in cooling conditions, is commensurate with the friction drag. Hence the gas cooler pressure drop is less compared with the evaporator pressure drop. Experimental results
show that the percentage of pressure drop in the gas cooler tube is very less compared to other refrigerant. This results lead to design possibilities of effective gas cooler. Experimental validation of Petrov-Popov correlation implies that the measured values are higher (up to 55% deviation) than predicted values. This deviation can be reduced by using Churchill equation instead of Filonenko equation.

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Reference